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The Construction of Explicit Warrant Derived from Implicit Warrant in Mathematical Proof

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Abstract. This study aims to describe how subjects construct explicit warrant derived from implicit warrant when completing mathematical proof. This research was conducted on seventeen students of mathematics education study programs by providing tests on vector material in elementary linear algebra courses. The test results show that there are four students who can solve the evidentiary questions correctly using warrant, but there is only one student who can be the subject of research. The selection of subjects is based on students' ability to communicate verbally about the thought process carried out in constructing implicit warrant to explicit warrant when conducting proof. Data in this study were obtained from think aloud, tests, and interviews conducted by researchers to subjects. The results showed that the explicit warrant constructed by the subjects was obtained from an implicit warrant based on the thought process carried out. The subject constructs an explicit warrant derived from an implicit warrant through four steps, namely identifying problems, determining warrant, doing algebraic manipulation, and making conclusions. Warrant is a guarantor used to get the correct conclusions from a mathematical proof, thus further research needs to be done related to warrant.

INTRODUCTION

Warrant is a component in proof of mathematics used to determine one's cognitive abilities. Warrant is the most important part in mathematics education to prove the truth of claims based on learning systems and cognitive processes carried out [1]. Claim is a statement that needs to be verified. A statement in proof of mathematics is a declarative sentence that is true or false, but it cannot be both [2][3]. Warrant in an argument can be interpreted as a guarantor used to find out the truth of the conclusions obtained.

Statements in mathematics need to be proven based on the structure of the argument formed. Arguments in mathematical proof are composed of premises and conclusions [4][5][6] .Arguments can be said as facilities or tools to construct one's knowledge based on the thought process carried out, if an individual is able to construct an argument then he understands the concepts used [7].

Based on the results of the study states that warrant is used to determine one's critical thinking ability through the process of evaluating, making decisions, and discussing the truth of the claim based on the construction of thought that is done [8]. Other research results also state that the process of constructing algebraic evidence can be carried out through five stages, namely reading propositions or statements that will be proven, evaluating truth, determining strategies, making plans, and thinking strategies [9]. While the results of research on the cognitive processes of students in constructing mathematics conjecture can be done through understanding the problem, exploring problems, formulating the conjecture, justifying the conjecture, and proving the conjecture [10]. Based on the results of previous

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studies it can be seen that there are no researchers who study about constructing warrant which appears implicitly to explicit warrant based on the process of cognition that is done, thus in this study examines the construction of explicit warrant derived from implicit warrant in a mathematical proof.

RESEARCH METHOD

This research was a descriptive exploratory study with a qualitative approach. In qualitative research, the existence of the researcher is very important, as a main instrument [11]. The study was conducted on seventeen students of the third semester of Mathematics Education study program in the subject of elementary algebra linear. The selection of subjects was done by looking at the ability of students to construct knowledge and their ability to communicate verbally about previous knowledge. There were four people out of seventeen students who are able to solve the evidentiary questions correctly, but there was only one student who can communicate verbally about the thought process carried out when proving.

Data collection in this study was obtained from tests, think aloud, and interviews. Interview and think aloud are used to explore the subject's ability to construct explicit warrant derived from implicit warrant because implicit warrant cannot be known if interview and think aloud are not conducted. Warrant implicitly arises based on the subject's thought process in completing mathematical proof. The test questions used in this study were:

If u and v are non zero vectors and θ is the angle between two vectors, then the product of u and v is defined as

$$u.v = \begin{cases} ||u||||v|| \cos\theta, & u \neq 0 \; and \; v \neq 0 \\ 0, & u = 0 \; and \; v = 0 \end{cases}$$

If u and v are two non-zero vectors as shown in Figure 1 and θ is the angle between u and v, so we applied cosine law $||\overline{MN}||^2 = ||u||^2 + ||v||^2 - 2||u||||v|| \cos\theta$.

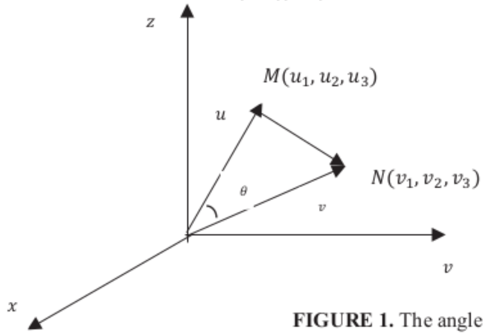


FIGURE 1. The angle between u and v

Prove that $u.v = u_1v_1 + u_2v_2 + u_3v_3 = u_1v_1 + u_2v_2 + u_3v_3$ based on information you know!

FINDING AND DISCUSSION

The results showed that the subjects construct explicit warrant which was derived from implicit warrant through the thought process carried out. The subject constructs the knowledge about the definitions that exist in the vectors and the previous theorem to carry out proof. The subject identified the problem by stating the information contained in the problem.

R : What information do you get from this problem?
S : The important information is the definition of $u.v$ and Cosine Law. And, I need to prove that $u.v = u_1v_1 + u_2v_2 + u_3v_3$

Furthermore, the subject used the definition of the distance between two points in the form $\overline{PQ} = v - u$ but, it changed to $\overline{MN} = v - u$ since it depends on the symbol of problem. Then the subject substitutes the cosines law in the problem, it obtained $||u||||v|| \cos\theta = \frac{1}{2}\left(||u||^2 + ||v||^2 - ||\overline{MN}||^2\right) = \frac{1}{2}\left(||u||^2 + ||v||^2 - ||u - v||^2\right)$.

The subject did proof on the three dimensional vector so he stated that it was necessary to use the norm definition of the vector $||\overline{P_1P_2}|| = ||u - v|| = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$, and it obtained $||u||||v|| \cos\theta = \frac{1}{2}\left(||u||^2 + ||v||^2 - (v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2\right)$. The subject stated that to elaborate $(v_1 - u_1)^2, (v_2 - u_2)^2$, dan $(v_3 - u_3)^2$ using the formula in the quadratic equation and the associative nature, but the subject stated that only during the interview but did not write down the worksheet.

R : What dimension vector do you proof?

S : Third dimension, Maam. So, I wrote $||u||^2 = u_1^2 + u_2^2 + u_3^2, ||v||^2 = v_1^2 + v_2^2 + v_3^2, ||v - u||^2 = (v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2$

R : $||u||^2 = u_1^2 + u_2^2 + u_3^2, ||v||^2 = v_1^2 + v_2^2 + v_3^2$? How did you obtain this?

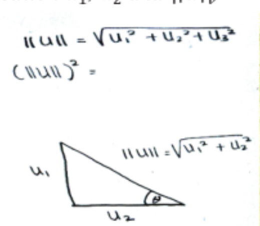
S : I referred to the second one, Maam

R : What do you mean?

S : In the second vector $||u||^2 = u_1^2 + u_2^2$ was obtained from the Pythagorean theorem Maam, so I added one component u_3 and v_3 for the third dimension

R : Could you explain the Pythagorean Theorem?

S : I confuse Maam" *(The subject drew a right triangle while doing think aloud to determine the location u_1, u_2 and $||u||$)*



So, here it is Maam…. The front side is u_1 and the bottom one is u_2 and it obtained hypotenuse $||u|| = \sqrt{u_1^2 + u_2^2}$. So, for the three dimension vector, just add one more component in the form u_3^2 and it obtained norm u was $||u|| = \sqrt{u_1^2 + u_2^2 + u_3^2}$, so is for vectors v.

The subject said that to describe norm u and norm v used the Pythagorean theorem, but he did not write it on the worksheet. Subjects did think aloud to get $||u||^2 = u_1^2 + u_2^2 + u_3^2$ by drawing a right triangle. He drew a right triangle with u_2 side and u_1 side, so that an oblique side $||u|| = \sqrt{u_1^2 + u_2^2}$ was obtained. The hypotenuse obtained by the subject was norm u in the two dimensional vector, then he used it for the three dimensional vector by adding one component in the form of u_3 and v_3. The subject wrote it because he remembered the definition of norm vector which stated that $||u|| = \sqrt{u_1^2 + u_2^2 + u_3^2}$ [12].

R : Then what is the quadratic equation used for?

S : In the quadratic equation the form is $(a - b)^2 = a^2 - 2ab + b^2$, so I sued it to elaborate $(v_1 - u_1)^2, (v_2 - u_2)^2$, and $(v_3 - u_3)^2$.

R : Then, how about the associative property?

S : I used it for algebraic manipulation to obtain $\frac{1}{2}(2u_1v_1 + 2u_2v_2 + 2u_3v_3) = \frac{1}{2}(2(u_1v_1 + u_2v_2 + u_3v_3)) = u.v$

The results of the interviews showed that the subjects gave a warrant that was not written in the worksheet but appeared during the interview and think aloud. This warrant was obtained from constructing knowledge about quadratic equations, Pythagorean Theorem, and the associative nature of algebraic manipulation. This is an implicit warrant raised by the subject to get an explicit warrant. Implicit warrants should be removed because students do not necessarily prove correctly if the warrant used cannot be known directly [13][14]. The activities carried out by subjects in constructing explicit warranties derived from implicit warranties in mathematical proof can be seen in Fig. 2.

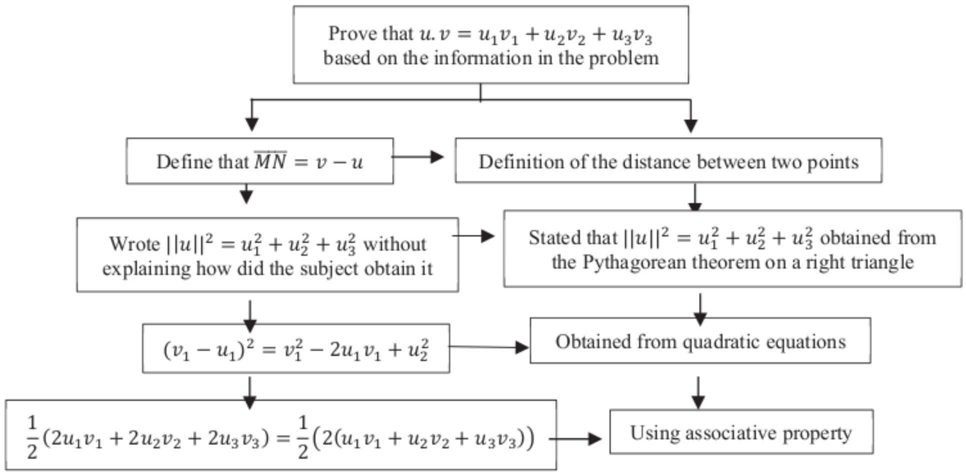


FIGURE 2. Subject's Thinking Construction

Based on the research findings, it can be found that the student's thought process in constructing explicit warrant derived from implicit warrant is carried out through four stages, namely: identifying questions, determining warrant, doing algebraic manipulation, and making conclusions and justifications. Hence, we attained differences about the steps of the student's thought process in research with previous theories. The difference can be seen in Table 1.

TABLE 1. Results Comparison with Previous Findings

Ozturk & Kaplan [9]	**Sternberg, Grigorenko and Zhang [15]**	**Current Research**
• Read the statement that will be proven. • Evaluate the truth. • Determine strategy. • Make a plan. • Thinking strategies	• Separation of objects from their context. • Tendency to focus on certain attributes. • Determination of categories. • Choosing on the use of rules	• Identifying problems • Determine warrant in the form of: vector definition, and Pythagorean theorem • Do algebraic manipulation • Make conclusions and justifications

Students identified problem as the first step to understand the problem proved [10]. Identifying the problem was done by mentioning if u dan v are two vectors in three dimensions completed by cosine rule. Then, students determine warrant in the form of distance definition of two points, norm vector definition, and pythagorean theorem. In this term, warrant means the rule or guarantor used by the students to solve mathematical proof [15]. Students manipulated algebra based on warrant. It was a strategic used to get conclusion and justification [9]. The thought process of the subject in constructing explicit warrants derived from implicit warrants was described more specifically through four steps. The four steps can be seen in Table 2.

TABLE 2: Detailed process of student cognition in constructing explicit warrant derived from implicit warrant.

Steps	Construction Process
Identification problem	• Stated that u and v is a nonzero vector in dimension three. • θ is an angle between two vectors. • Cosine law.
Determining warrant	• Using the definition of the distance between two points of $\overline{P_1P_2}$ then it changed into $\overline{MN}$ since it followed the information in the problem. • Defining norm vector. • Using Pythagorean Theorem to obtain $\|\|u\|\|^2 = u_1^2 + u_2^2 + u_3^2$ and $\|\|u\|\|^2 = v_1^2 + v_2^2 + v_3^2$.
Perform algebraic manipulation	• Using quadratic equation to elaborate $(v_1 - u_1)^2$, $(v_2 - u_2)^2$, and $(v_3 - u_3)^2$. • Using associative property to obtain $u.v = \frac{1}{2}(2u_1v_1 + 2u_2v_2 + 2u_3v_3) = \frac{1}{2}\left(2(u_1v_1 + u_2v_2 + u_3v_3)\right) = u_1v_1 + u_2v_2 + u_3v_3$
Concluding	• Observe every step of the evidence starting from the information known to the problem until it obtained $u.v = u_1v_1 + u_2v_2 + u_3v_3$. • Justifying that the proof is correct.

CONCLUSION

Warrant can be interpreted as a guarantor used to find out the truth of the conclusion of an argument. Arguments in mathematical proof consist of premises and conclusions. Warrants that arise in mathematical proof can occur implicitly and explicitly. Warrant explicitly appear directly at the time of proof, so it can be seen that the guarantor used to prove is true or not. While implicit warrant does not appear directly when evidenced but appears during think aloud and interview, hence implicit warrant must be removed because the truth of the guarantor used cannot be known directly. The process of thinking of students in constructing explicit warrants derived from implicit warrants in completing mathematical proof can be done through four steps, namely identifying questions, determining warrant, doing algebraic manipulation, and making conclusions. Each step is carried out by students to get the correct conclusions from the proof process carried out.

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